Universidad Autónoma de Baja California
Facultad de Ingeniería, Arquitectura y Diseño

INGENIERÍA EN NANOTECNOLOGÍA

ETAPA DISCIPLINARIA

TAREAS

13185 TEORÍA ELECTROMAGNÉTICA

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- TEORÍA ELECTROMAGNÉTCA -

Tarea 1
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1. Two uniform line charges of $\rho_\ell = 4$ nC/m each are parallel to the z axis at $x = 0$, $y = \pm 4$ m. Determine the electric field $\vec{E}$ at $(\pm 4, 0, z)$.

2. Determine $\vec{E}$ at the origin due to a uniform line charge distribution with $\rho_\ell = 3.30$ nC/m located at $x = 3$ m, $y = 4$ m.

3. The plane $-x + 3y - 6z = 6$ contains a uniform charge distribution $\rho_S = 0.53$ nC/m^2. Find $\vec{E}$ on the side containing the origin.

4. Two infinite sheets of uniform charge density $\rho_S = 10^{-9}/6\pi$ C/m^2 are located at $z = -5$ m and $y = -5$ m. Determine the uniform line charge density ρ_ℓ necessary to produce the same value of $\vec{E}$ at $(4, 2, 2)$ m, if the line charge is located at $z = 0$, $y = 0$.

5. A circular ring of charge with radius 2 m lies in the $z = 0$ plane, with center at the origin. If the uniform charge density is $\rho_\ell = 10$ nC/m, find the point charge Q at the origin which produce the same electric field $\vec{E}$ at $(0, 0, 5)$ m.

6. A circular disk $r \leq 2$ m in the $z = 0$ plane has a charge density $\rho_S = 10^{-8}/r$ C/m^2. Determine the electric field $\vec{E}$ for the point $(0, \phi, h)$.

7. A circular disk $r \leq 1$ m, $z = 0$ has a charge density $\rho_S = 2(r^2 + 25)^{3/2} e^{-10r}$ C/m^2. Find $\vec{E}$ at $(0, 0, 5)$ m.

8. Two uniform charge distributions are as follows: a sheet of uniform charge density $\rho_s = -50$ nC/m^2 at $y = 2$ m and a uniform line of $\rho_\ell = 0.2$ μC/m at $z = 2$ m, $y = -1$ m. At what points in the region will $\vec{E}$ be zero?

9. A finite sheet of charge, of density $\rho_s = 2x(x^2 + y^2 + 4)^{3/2}$ (C/m^2), lies in the $z = 0$ plane for $0 \leq x \leq 2$ m and $0 \leq y \leq 2$ m. Determine $\vec{E}$ at $(0, 0, 2)$ m.

10. Charge is distributed with constant density ρ_v throughout a spherical volume of radius a. Show that

$$\vec{E} = \begin{vmatrix} \frac{r\rho_v}{3\epsilon_0}\hat{a}_r \; ; \; r \leq a \\ \\ \frac{a^3\rho_v}{3\epsilon_0 r^2}\hat{a}_r \; ; \; r \geq a \end{vmatrix}$$

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Tarea 2
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1. Charge is distributed in the spherical region $r \leq 2$ m with density $\rho = \frac{-200}{r^2}$ (μC/m^3). What net flux crosses the surfaces $r = 1$ m, $r = 4$ m, $r = 500$ m?

2. If a point charge Q is at the origin, find an expression for the flux which crosses the portion of a sphere, centered at the origin, described by $\alpha \leq \phi \leq \beta$.

3. A uniform line charge with $\rho_\ell = 3$ μC/m lies along the x axis. What flux crosses a spherical surface centered at the origin with $r = 3$ m?

4. Given that $\vec{D} = 500e^{-0.1x}\,\hat{a}_x$ (μC/m^2), find the flux Ψ crossing surfaces of area 1 m^2 normal to the x axis located at $x = 1$ m, $x = 5$ m, and $x = 10$ m.

5. Given a charge distribution with density $\rho = 5r$ (C/m^3) in spherical coordinates, use Gauss's law to find $\vec{D}$.

6. Given $\vec{D} = \frac{10}{r^2}[1 - e^{-2e}(1 + 2r + 2r^2)]\,\hat{a}_r$ in spherical coordinates, find the charge density.

7. In the region $a \leq r \leq b$ (cylindrical coordinates) $\vec{D} = \rho_0(\frac{r^2-a^2}{2r})\,\hat{a}_r$, and for $r > b$, $\vec{D} = \rho_0(\frac{b^2-a^2}{2r})\,\hat{a}_r$. For $r < a$, $\vec{D} = 0$. Find ρ in all three regions.

8. Given $\vec{D} = (\frac{5r^2}{4})\,\hat{a}_r$ (C/m^2), in spherical coordinates, evaluate both sides of the divergence theorem for the volume enclosed by $r = 4$ m and $\theta = \pi/4$.

9. Given that $\vec{D} = 2r\cos(\phi)\,\hat{a}_\phi - \frac{\sin(\phi)}{3r}\,\hat{a}_z$ in cylindrical coordinates, find the flux crossing the portion of the $z = 0$ plane defined by $r \leq a$, $0 \leq \phi \leq \pi/2$. Repeat for $3\pi/2 \leq \phi \leq 2\pi$. Assume flux is positive in the $\hat{a}_z$ direction.

10. A point charge, $Q = 2000$ pC, is at the origin of spherical coordinates. A concentric spherical distribution of charge at $r = 1$ m has a charge density $\rho_s = 40\pi$ pC/m^2. What surface charge density on a concentric shell at $r = 2$ m would result in $\vec{D} = 0$ for $r > 2$ m?

11. An electrostatic field is given by $\vec{E} = \lambda(x\,\hat{a}_x + y\,\hat{a}_y)$ where λ is a constant. Use Gauss's law to find the total charge enclosed by the surface consisting of S_1, the curved portion of the half-cylinder $z = (r^2 - y^2)^{1/2}$ of length h; the two semi circular plane end pieces, S_2 and S_3; and S_4 the rectangular portion of the xy-plane. Express your result in terms of λ, r and h.

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Tarea 3
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1. An electrostatic field is given by $\vec{E} = (\frac{x}{2} + 2y)\,\hat{a}_x + 2x\,\hat{a}_y$. Find the work done in moving a point charge $Q = -20\mu$C
   (a) From the origin to $(4, 0, 0)$ m.
   (b) From $(4, 0, 0)$ m to $(4, 2, 0)$ m.
   (c) From $(4, 2, 0)$ m to $(0, 0, 0)$ m.

2. Two point charges -4μC and 5μC are located at $(2, -1, 3)$ and $(0, 4, -2)$, respectively. Find the potential at $(1, 0, 1)$.

3. A total charge of 40/3 nC is uniformly distributed in the form of a circular disk of radius 2m. Find the potential due to the charge at point of the axis, 2m from the disk.

4. A point charge of 5 nC is located at $(-3, 4, 0)$, while line $y = 1$, $z = 1$ carries uniform charge 2 nC/m.
   (a) If $V = 0$ V at $O(0, 0, 0)$, find V at $A(5, 0, 1)$.
   (b) If $V = 100$ V at $B(1, 2, 1)$, find V at $C(-2, 5, 3)$.
   (c) If $V = -5$ V at O, find V_{BC}.

5. Given the electric field $\vec{E} = 2x\,\hat{a}_x - 4y\,\hat{a}_y\ (\frac{V}{m})$, find the work done in moving a point charge +2 C
   (a) from $(2, 0, 0)$ m to $(0, 0, 0)$ m and then from $(0, 0, 0)$ m to $(0, 2, 0)$ m;
   (b) from $(2, 0, 0)$ m to $(0, 2, 0)$ m along the straight-line path joining the two points.

6. Given the field $\vec{E} = (k/r)\,\hat{a}_r\ (\frac{V}{m})$ in cylindrical coordinates, show that the work needed to move a point charge Q from any radial distance r to a point at twice that radial distance is independent of r.

7. For a line charge $\rho_\ell = (10^{-9}/2)$ C/m on the z axis, find V_{AB}, where A is $(2\,\text{m}, \pi/2, 0)$ and B is $(4\,\text{m}, \pi, 5\,\text{m})$.

8. Given the field $\vec{E} = (-16/r^2)\,\hat{a}_r\ (\frac{V}{m})$ in spherical coordinates, find the potential of point $(2\,\text{m}, \pi, \pi/2)$ with respect to $(4\,\text{m}, 0, \pi)$

9. Find the work done in moving a point charge $Q = -20\ \mu$C from the origin to $(4, 2, 0)$ m in the field $\vec{E} = 2(x + 4y)\,\hat{a}_x + 8x\,\hat{a}_y\ (\frac{V}{m})$ along the path $x^2 = 8y$.

10. Find the work done in moving a point charge $Q = 3\mu$C from $(4\text{m}, \pi, 0)$ to $(2\text{m}, \pi/2, 2\text{m})$, cylindrical coordinates, in the field $\vec{E} = (10^5/r)\,\hat{a}_r + (10^5)z\,\hat{a}_z\ (\frac{V}{m})$.

11. Find the difference in the amounts of work required to bring a point charge $Q = 2$ nC from infinity to $r = 2$ m and from infinity to $r = 4$ m, in the field $\vec{E} = (10^5/r)\,\hat{a}_r\ (\frac{V}{m})$.

12. A uniform line charge of density $\rho_\ell = 1$ nC/m is arranged in the form of a square 6 m on a side. Find the potential at $(0, 0, 5)$ m.

13. A total charge of 160 nC is first separated into four equal point charges spaced at 90^0 intervals around a circle of 3 m radius. Find the potential on the axis, 5 m from the plane of the circle. Separate the total charge into eight equal parts and repeat with the charges at 45^0 intervals. What would be the answer in the limit $\rho_\ell = (160/6\pi)$ nC/m?

14. A uniform line charge $\rho_\ell = 2$ nC/m lies in the $z = 0$ plane parallel to the x axis at $y = 3$ m. Find the potential difference V_{AB} for the points $A(2, 0, 4)$ m and $B(0, 0, 0)$.

15. A uniform sheet of charge, $\rho_s = (1/6\pi)$ nC/m^2, is at $x = 0$ and a second sheet, $\rho_s = (-1/6\pi)$ nC/m^2, is at $x = 10$ m. Find V_{AB}, V_{BC}, and V_{AC} for $A(10\,\text{m}, 0, 0)$, $B(4\,\text{m}, 0, 0)$, and $C(0, 0, 0)$ m.

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Tarea 4

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1. A conducting filament carries current I from point $A(0,0,a)$ to point $B(0,0,b)$. Show that at the point $P(x,y,0)$, $\vec{H} = \frac{1}{4\pi\sqrt{x^2+y^3}}[\frac{b}{\sqrt{x^2+y^2+b^2}} - \frac{a}{\sqrt{x^2+y^2+a^2}}]\,\hat{a}_\phi$

2. Consider AB in Figure 1 as part of an electric circuit. Find $\vec{H}$ at the origin due to AB.

3. (a) Find $\vec{H}$ at $(0,0,5)$ due to side 2 of the triangular loop in Figure 2.
   (b) Find $\vec{H}$ at $(0,0,5)$ due to entire loop.

4. A square conducting loop of side $2a$ lies in the $z=0$ plane and carries a current I in the counterclockwise direction. Show that at the center of the loop $\vec{H} = \frac{\sqrt{2}I}{\pi a}\,\hat{a}_z$.

5. A thin ring of radius 5 cm is placed on plane $z=1$ cm so that its center is at $(0,0,1)$ cm. If the ring carries 50 mA along $\hat{a}_\phi$, find $\vec{H}$ at
   $(a)(0,0,-1)$ cm
   $(b)(0,0,10)$ cm.

6. (a) A filament loop carrying current I is bent to assume the shape of a rectangular polygon of n sides. Show that at the center of the polygon $\vec{H} = \frac{nI}{2\pi r}\sin\frac{\pi}{n}$, where r is the radius of the circle circumscribed by the polygon.
   (b) Apply this for the case of $n=3$ and $n=4$.
   (c) As n become large, show that the result of part (a) becomes that of the circular loop $\vec{H} = \frac{I\rho^2}{2[\rho^2+h^2]^{3/2}}\,\hat{a}_z$

7. A solenoid has 2000 turns, a length of 75 cm, and a radius of 5 cm. If it carries a current of 50 mA along $\hat{a}_\phi$, find $\vec{H}$ at
   $(a)(0,0,0)$ cm
   $(b)(0,0,75)$ cm
   $(c)(0,0,50)$ cm.

8. Plane $y=1$ carries current $\vec{K} = 50\,\hat{a}_z$ mA/m. Find $\vec{H}$ a t
   $(a)(0,0,0)$
   $(b)(1,5,-3)$.

9. Two identical current loops have their centers at $(0,0,0)$ and $(0,0,4)$ and their axes the same as the z-axis (so the -Helmholtz coil- is formed). If each loop has a radius of 2 m and carries a current of 5 A in $\hat{a}_\phi$, calculate $\vec{H}$ at
   $(a)(0,0,0)$
   $(b)(0,0,2)$.

10. If $\vec{H} = y\,\hat{a}_x - x\,\hat{a}_y$ A/m on plane $z = 0$, (a) determine the current density and (b) verify Ampèr's law by taking circulation of $\vec{H}$ around the edge of rectangle $z = 0, 0 < x < 3, -1 < y < 4$.

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Tarea 5

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1. Show that the magnetic field due to the finite current element shown in Fig. 1 is given by $\vec{H} = \frac{I}{4\pi r}(\sin\alpha_1 - \sin\alpha_2)\,\hat{a}_\phi$.

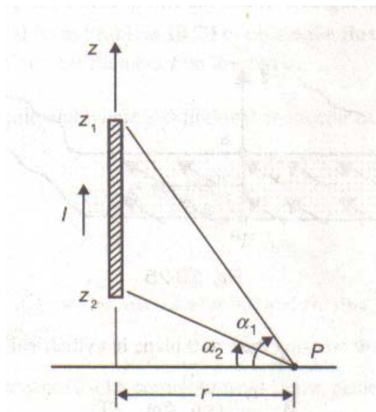

Figure 1: Finite current element

2. Obtain $d\vec{H}$ at a general point (r, θ, ϕ) in spherical coordinates, due to a differential current element $Id\vec{\ell}$ at the origin in the positive z direction.

3. Currents in the inner and outer conductors of Fig. 2 are uniformly distributed. Use Ampère's law to show that for $b \le r \le c$, $\vec{H} = \frac{I}{2\pi r}\left(\frac{c^2-r^2}{c^2-b^2}\right)\hat{a}_\phi$.

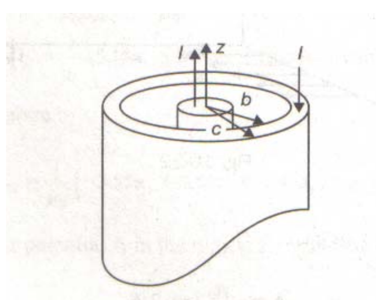

Figure 2: Currents in the inner and outer conductors

4. A current filament of 10 A in the $+y$ direction lies along the y axis, and a current sheet, $\vec{K} = 2.0\,\hat{a}_x$ A/m, is located at $z = 4$ m. Determine $\vec{H}$ at the point $(2, 2, 2)$.

5. A cylindrical conductor of radius 10^{-2} m has an internal magnetic field $\vec{H} = (4.77 \times 10^4)(\frac{r}{2} - \frac{r^2}{3\times10^{-2}})\,\hat{a}_\phi$ (A/m). What is the total current in the conductor?

6. In cylindrical coordinates, $\vec{J} = 10^5(\cos^2 2r)\,\hat{a}_z$ in a certain region. Obtain $\vec{H}$ from this current density and then take the curl of $\vec{H}$ and compare with $\vec{J}$.

7. In Cartesian coordinates, a constant current density, $\vec{J} = J_0\,\hat{a}_y$, exist in the region $-a \leq z \leq a$. See Fig. 3. Use Ampère's law to find $\vec{H}$ in all regions. Obtain the curl of $\vec{H}$ and compare with $\vec{J}$.

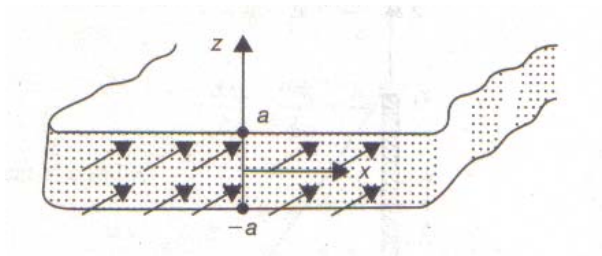

Figure 3: Region $-a \leq z \leq a$

8. Given that the vector magnetic potential within a cylindrical conductor of radius a is $\vec{A} = -\frac{\mu_o I r^2}{4\pi a^2}\,\hat{a}_z$ find the corresponding $\vec{H}$.

9. One uniform current sheet, $\vec{K} = K_0\,(-\hat{a}_y)$, is located at $x = 0$ and another, $\vec{K} = K_0\,\hat{a}_y$, is at $x = a$. Find the vector potential between the sheets.

10. One uniform sheet, $\vec{K} = K_0\,\hat{a}_y$, is at $z = b > 2$ and another, $\vec{K} = K_0\,(-\hat{a}_y)$, is at $z = -b$. Find the magnetic flux crossing the area defined by $x =$ const,, $-2 \leq x \leq 2$, $0 \leq y \leq L$. Assume free space.